Int. J. Multiphase Flow Vol. 19, No. 5, pp. 829–838, 1993
Printed in Great Britain.

0301-9322/93 $6.00 + 0.00

A MODEL FOR SLUG LENGTH DISTRIBUTION IN GAS–LIQUID SLUG FLOW

D. BARNEA and Y. TAITEL
Department of Fluid Mechanics and Heat Transfer, Faculty of Engineering, Tel-Aviv University, Ramat-Aviv 69978, Israel

(Received 17 January 1993; in revised form 30 May 1993)

Abstract—Intermittent, or slug flow, is a very common occurrence in gas–liquid two-phase pipe flow. Usually slug flow is an undesirable flow pattern since the existence of long lumps of liquid slugs that move at high speed is unfavorable to gas–liquid transportation. Considerable efforts have been devoted to the prediction of the slug hydrodynamic characteristics, primarily by considering an average slug length and calculating average parameters. This approach is useful, and in many cases it is adequate for many engineering calculations. There are, however, cases where this information is not sufficient and much more detailed information concerning the slug length distribution, the mean slug length and the maximum possible slug length is essential. This work presents a model that is able to calculate the slug length distribution at any desired position along the pipe. The model assumes a random distribution at the inlet of the pipe and it calculates the increase or decrease in each individual slug length, including the disappearance of the short slugs, as they move downstream. The results of the calculation show that for the fully developed slug flow the mean slug length is about 1.5 times the minimum stable slug length and the maximum length is about 3 times the minimum stable slug length.

Key Words: two-phase, slug-flow, slug-length

INTRODUCTION

The main characteristics of gas–liquid slug flow are intermittency and irregularity. Each slug unit is composed of a liquid slug containing dispersed bubbles and an elongated bubble zone, which has a stratified or annular configuration, depending on the tube inclination. Due to the unsteady and developing character of slug flow, flow parameters such as the length of the liquid slug and the length of the elongated bubbles should be described in statistical terms. Knowledge of the time-averaged values of these quantities is not always sufficient and information regarding the length distribution is often essential. Of particular importance is the maximum possible slug length, since slug catchers design depends on the longest encountered slug and not necessarily on the average one. It is also important to know the development of the slug length distribution along the pipe; namely, the variation of the corresponding length distribution close to the pipe entrance, as well as in the fully developed region further downstream.

Slug length and slug frequency are interrelated properties and often are used alternatively. Experimental observations for air–water systems in upward vertical and horizontal flows suggest that the average stable liquid slug length is relatively insensitive to the gas and liquid flow rates and depends mainly on the pipe diameter. The average slug length has been observed to be about 15–40 pipe diameters for horizontal flow (Dukler & Hubbard 1975; Nicholson *et al.* 1978; Barnea & Brauner 1985; Fabre & Liné 1992) and 8–25 pipe diameters for vertical flow (Moissis & Griffith 1962; Moissis 1963; Akagawa & Sakaguchi 1966; Fernandes 1981; Barnea & Shemer 1989).

Slug frequency has sometimes been considered as an entrance phenomenon, namely, it results from the bridging of the liquid at the entrance (Taitel & Dukler 1977). This is indeed the case in horizontal and slightly inclined flows, near the transition from stratified flow. In this case low-frequency slugs are generated, causing relatively long liquid slugs at the entrance, which propagate downstream. However, generally, short (high-frequency) slugs are formed at the entrance of the pipe. These slugs are usually unstable. Shedding of liquid at the rear of the liquid slug seems to be larger for short slugs. As a result, an elongated bubble behind a short slug moves faster and overtakes the bubble ahead of it (Moissis & Griffith 1962). The bubble and the corresponding liquid slug merge in this process, decreasing the slug frequency. The merging process continues until the liquid slug is long enough to be stable, namely when the trailing bubble is unaffected by the wake of the leading one. This occurs when the velocity profile at the rear of the liquid slug can be considered fully developed (Moissis & Griffith 1962; Taitel *et al.* 1980; Barnea & Brauner 1985; Dukler *et al.* 1985).

Moissis & Griffith (1962), Taitel *et al.* (1980) and Barnea & Brauner (1985) simulated the mixing process between the film and the slug by a wall jet entering a large reservoir. It was suggested that a developed slug length is equal to the distance at which the jet has been absorbed by the liquid. Dukler *et al.* (1985), on the other hand, solved the boundary layer equations for calculating the developed slug length. Although the two approaches are different, the final prediction of the average slug length is similar. Shemer & Barnea (1987) detected the velocity field in the wake of the bubble by using the hydrogen bubble technique and utilized the results for estimating the minimum stable slug length. All the above approaches provide an estimation of the average stable slug length and tell nothing about the slug length distribution and the maximum possible slug length. As is well known, the slug length is widely dispersed around its average (Fabre & Liné 1992). Van Hout *et al.* (1992) measured the slug length distribution in upward vertical flow and found that the ratio between the standard deviation and the average is within 20–40%. To date, little information is available about the slug length distribution, although some interesting approaches have been introduced recently.

Brill *et al.* (1981), based on data from the Prudhoe bay field, were the first to suggest that the slug length distribution follows a log-normal distribution for large-diameter pipes. Nydal *et al.* (1992) measured the statistical distributions of some slug characteristics in an air–water horizontal system. They showed that the cumulative probability density function of the measured slug lengths fits a log-normal distribution well. Bernicot & Drouffe (1989) proposed a probabilistic approach for slug formation at the entrance of a horizontal pipe. They also modeled the evolution of the length distribution by an individual equation for each slug. Their approach is based on the concept that shedding for short slugs is greater than that for long slugs. Saether *et al.* (1990) analyzed data from different horizontal two- and three-phase pipeline systems and concluded that the liquid slug length distribution obeys fractal statistics. Dhulesia *et al.* (1991) used a 1-D Brownian motion with drift theory to obtain the slug length distribution.

In the present work a model for the slug length distribution at various positions along the pipe is suggested. The model is based on the bubble overtaking mechanism which occurs when the liquid slugs are shorter than the stable developed slug length.

MODEL FOR THE SLUG LENGTH DISTRIBUTION

The model assumes that short liquid slugs are generated at the entrance of the pipe. The generated slug lengths at the entrance are distributed randomly. It will be shown later that the results—namely the evolution of the length distribution along the pipe, the fully developed distribution, the average slug length, the maximal slug length and the standard deviation—are not sensitive to the nature of the initial distribution. Two kinds of entrance distributions have been chosen: the first is a random number set with a uniform distribution within a prescribed range; and the second is a normal distribution with a prescribed average and standard deviation.

The process of the evolution of the slug length distribution and the establishment of a stable developed distribution can be visualized as follows. Figure 1 shows a schematic structure of the slugs as they are generated at the entrance and move downstream. The propagation velocity of a trailing bubble is related to the maximum instantaneous liquid velocity ahead of it (Moissis & Griffith 1962; Shemer & Barnea 1987). The velocity field behind a leading bubble changes from an annular jet flow pattern in the near-wake region to fully developed pipe flow far away from the leading bubble. Thus, the value of the maximum instantaneous liquid velocity decreases with the distance from the leading bubbles. Therefore, bubbles behind short slugs travel much faster than bubbles behind long slugs (Moissis & Griffith 1962; Shemer & Barnea 1987). The bubbles are accelerated in the wake region, while their velocity reduces exponentially as the separation distance between the leading and trailing bubbles increases. Since the slug lengths (l_{s_i}) generated at the pipe

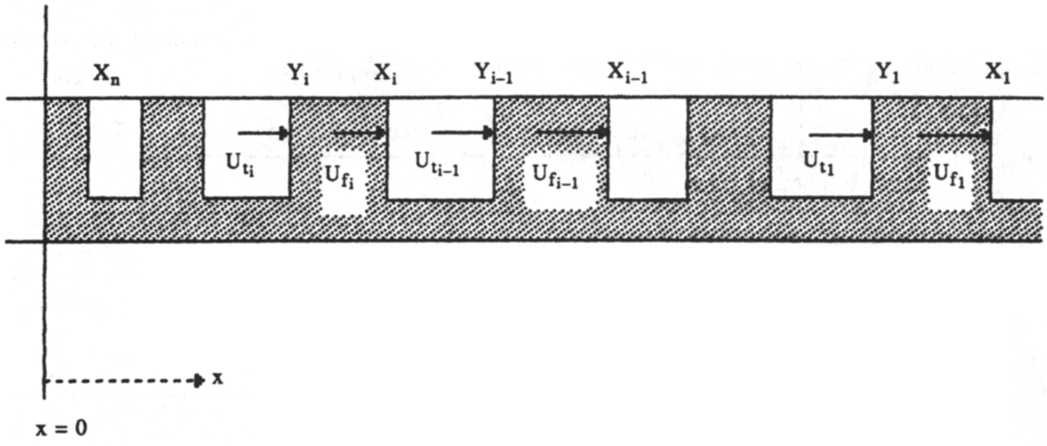

Figure 1. Schematic slug distribution at the entrance.

entrance are of different lengths, which are randomly distributed, the elongated bubbles behind these liquid slugs propagate at different velocities, such that $U_{t_i} = f(l_{s_i})$.

Trailing bubbles that are faster than the leading ones will overtake the leading bubbles. During the merging process the liquid slug length as well as the bubble length increase. The process of overtaking is terminated once all the slugs are long enough such that the velocity profile at the back of the slugs is fully developed and all the bubbles propagate at the same velocity.

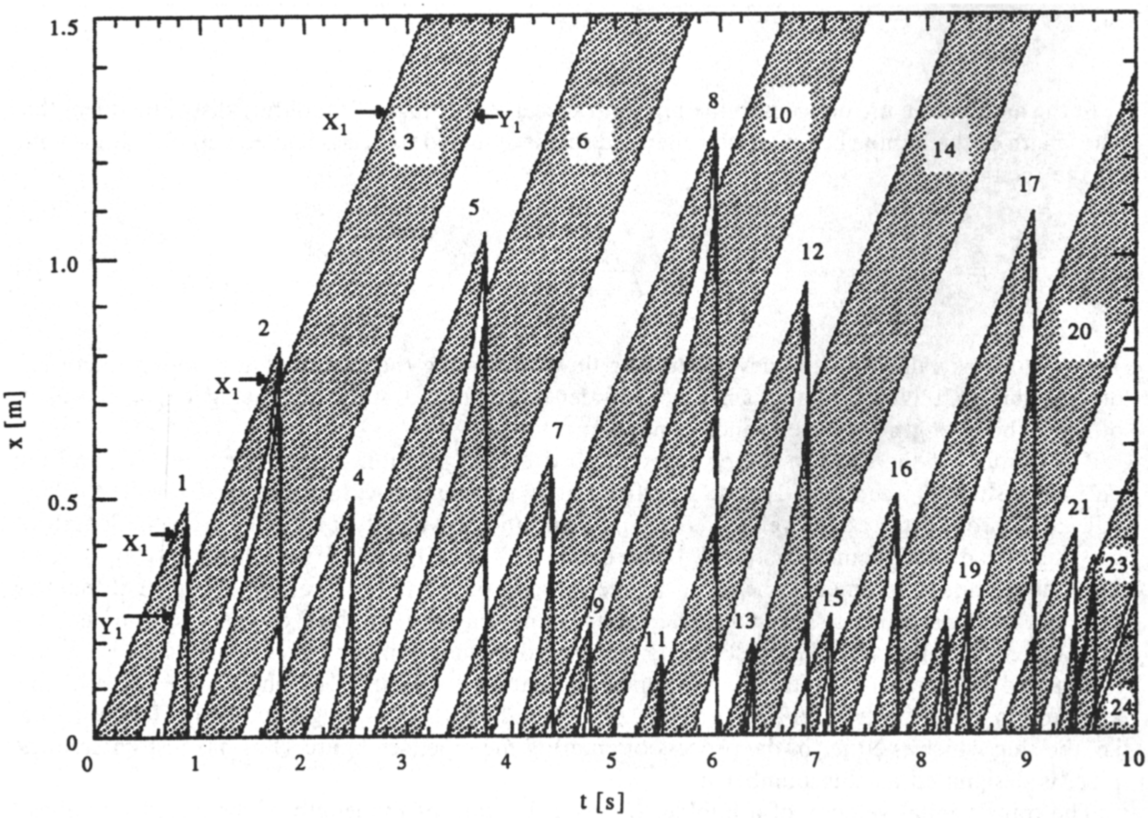

Figure 2. Slug generation and motion near the entrance.

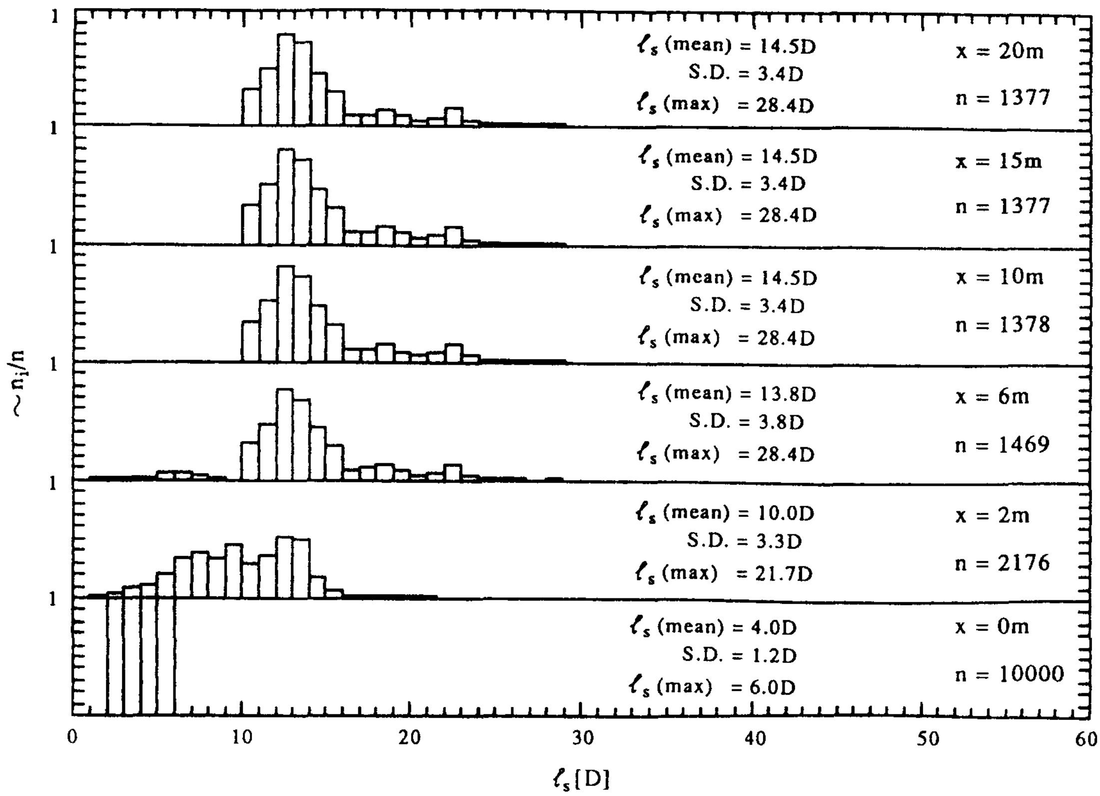

Figure 3. Slug length distribution ($U_{LS} = 0.01$ m/s, $U_{GS} = 0.25$ m/s); uniform random input.

In the analysis we introduce into the pipe entrance a set of slugs of randomly distributed lengths. The length of the bubble behind each generated slug is assumed to be associated with the slug length by the relation

$$\frac{l_b}{l_b + l_s} = \frac{U_{GS}}{U_t}. \quad [1]$$

This relation is valid for fully developed slug flow, neglecting the film thickness and the aeration of the liquid slug. Note also that since we consider the evolution of the liquid slug length, the effect of the bubble length at the entrance is not important.

The motion of the slugs in the pipe is described by the position of the slug front, X_i, and the tail of the slug, Y_i (see figure 1). The slug front at X_i propagates with a velocity U_{f_i} while the slug tail, at Y_i propagates at the velocity U_{t_i}. The translational velocity U_{t_i} depends on the length of the slug ahead of it, namely $U_{t_i} = f(l_{s_i})$ where $l_{s_i} = X_i - Y_i$. The velocity of the slug front, U_{f_i}, on the other hand, is assumed to be equal to the velocity of the tail of the slug ahead of it, namely $U_{f_i} = U_{t_{i-1}}$. This is consistent with the assumption that the bubbles do not deform as they propagate along the pipe. The first slug is designated as slug number 1 and its front velocity, U_{f_1}, is assumed to be equal to the fully developed translational velocity. The physical meaning of this assumption is that in front of the first slug one has a hypothetical long liquid slug. The last slug, i.e. the slug which is either in the process of entering the pipe (see figure 1) or has just entered the pipe, is designated as slug number n.

The translational velocity of a bubble, U_{t_i}, as a function of the length of the liquid slug ahead of it, should be given as an input relation. Moissis & Griffith (1962) measured the rise velocity of trailing bubbles behind a long bubble in vertical flow and expressed this velocity as a function of the separation distance between the leading and trailing bubbles. They suggested an exponential relation of the form

$$U_t = U_{t_\infty}\left[1 + 8\exp\left(-1.06\frac{l_s}{D}\right)\right], \qquad [2]$$

where D is the pipe diameter and l_s is the slug length ahead of the bubble. This equation is an average result for data taken primarily at low liquid and gas flow rates and pipe diameters in the range 1.2–5 cm. We adopted their format and used the following expression:

$$U_t = U_{t_\infty}\left[1 + B\exp\left(-\beta\frac{l_s}{l_{stab}}\right)\right], \qquad [3]$$

where l_{stab} is the minimum length of a stable slug and U_{t_∞} is the translational velocity of the bubble behind a long liquid slug, i.e. a slug that is longer than the minimum stable length l_{stab}. This velocity is traditionally correlated in the form

$$U_{t_\infty} = CU_s + U_d, \qquad [4]$$

where U_s is the mixture velocity, equal to

$$U_s = U_{LS} + U_{GS}, \qquad [5]$$

C is a parameter which equals about 1.2 and U_d is the drift velocity, namely the velocity of a bubble in stagnant liquid. Note that for a slug length l_s longer than l_{stab}, U_t is a constant equal to U_{t_∞}.

The motion of each slug is described by the change of the positions of X_i and Y_i with time and is given by

$$Y_i^{t+\Delta t} = Y_i^t + U_{t_i}^t \Delta t \qquad [6]$$

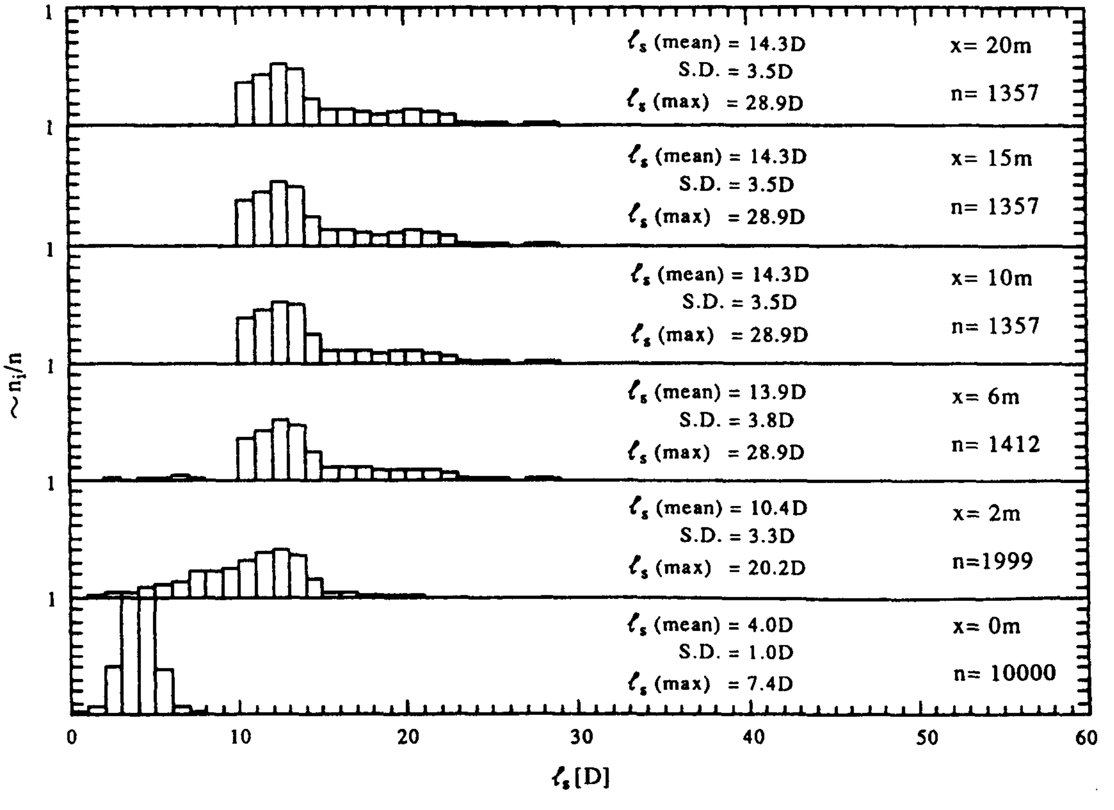

Figure 4. Slug length distribution ($U_{LS} = 0.01$ m/s, $U_{GS} = 0.25$ m/s); normal random input.

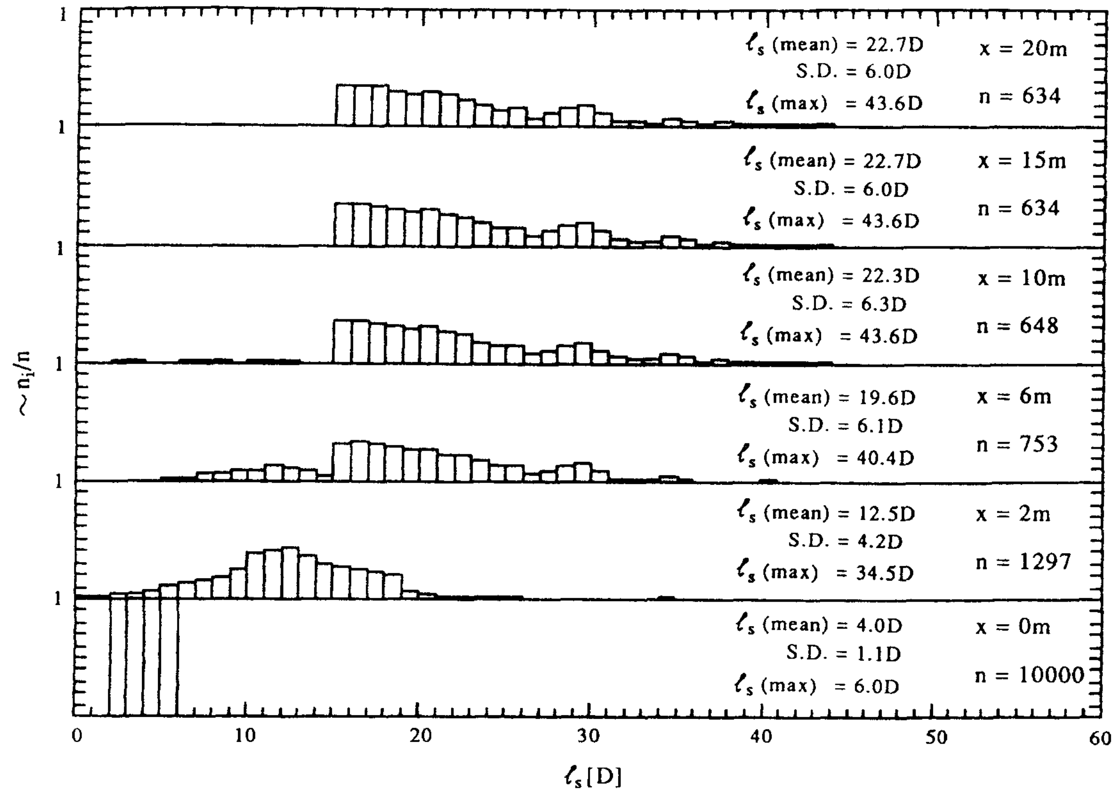

Figure 5. Slug length distribution ($U_{LS} = 0.75$ m/s, $U_{GS} = 0.75$ m/s); uniform random input.

and

$$X_i^{t+\Delta t} = X_i^t + U_{f_i}^t \Delta t, \qquad [7]$$

where $U_{t_i} = U_{t_i}(X_i - Y_i)$ and $U_{f_i} = U_{t_{i-1}}$.

Figure 2 demonstrates how the procedure works. In this figure the x positions of the slug front, X_i, and the slug tail, Y_i, are plotted vs time. The evolution of the 24 slugs that entered the pipe at a random length chosen between 2–6 pipe diameters is shown. The liquid slug zone is designated by the shaded area, while the elongated bubbles are the clear area in between the liquid slugs. Along the abscissa ($x = 0$) one can see the time when the slug front as well as the slug tail enter the pipe. The first slug to enter the pipe is designated in this figure as slug 1, the second as slug 2 etc. The increase or the decrease in the liquid slug length as it moves downstream depends on the relative velocity between its front and its back, namely $U_{t_i} - U_{f_i}$. When this value is positive the slug length decreases, and vice versa. Since $U_{f_i} = U_{t_{i-1}}$ and since the translational velocity U_{t_i} depends on the slug length, short slugs behind longer ones tend to decrease in length while slugs behind shorter slugs grow in length. As a result, short slugs whose lengths decrease eventually disappear and their liquid is accumulated in the slug behind them. This process can be clearly observed in figure 2. In this figure slugs Nos 1 and 2 disappear after <2 s. The liquid of slug No. 1 is first accumulated into slug No. 2 and then the liquid from slug No. 2 accumulates in slug No. 3. As mentioned previously, short slugs behind long slugs tend to disppear. For example, the front velocity of slug No. 4 is relatively low [the slope of the line $X(t)$] because it is behind a relative long slug (No. 3). The velocity of the slug back, however, increases as the slug length decreases. In this process slug No. 4 becomes shorter and disappears, i.e. it is merged with the slug behind it, namely slug No. 5. The slug numbers designated in figure 2 are the numbers of the slugs that enter the pipe and not the numbers of the slugs in the calculation procedure. In the calculation process, when a slug disappears its number becomes the number of the slug behind it and all the slug numbers

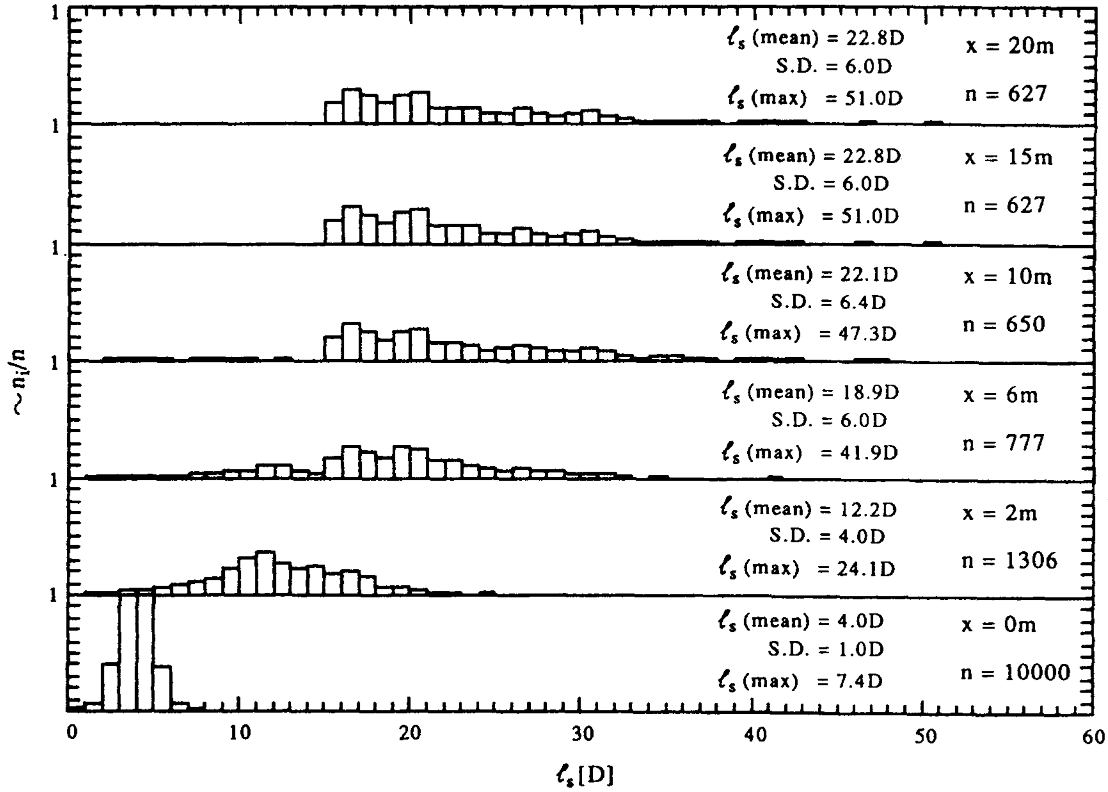

Figure 6. Slug length distribution ($U_{LS} = 0.75$ m/s, $U_{GS} = 0.75$ m/s); normal random input.

behind the disappearing slug decrease by one. This is reflected in figure 2 by the vertical lines, which appear whenever a liquid slug disappears, which indicate a reshuffling of the slug numbers in the computer. Thus, for example, slug No. 1 is designated by X_1 and Y_1. But when it disappears X_2 becomes X_1 and Y_2 becomes Y_1. Likewise, slug No. 3 is designated by X_1 and Y_1 when slugs Nos 1 and 2 disappear. An interesting point to observe is that short slugs are followed by very short bubbles. The reason for this is that for short slugs the translational velocity, U_t, is high. As a result, the bubble length calculated by [1] is short.

Figure 2 shows only the beginning of the calculation procedure for 10 s during which only 24 slugs entered the pipe. Out of the 24 slugs shown, 17 are seen to disappear within a very short distance of < 1.5 m from the entrance. In this process fewer, albeit longer, slugs are generated.

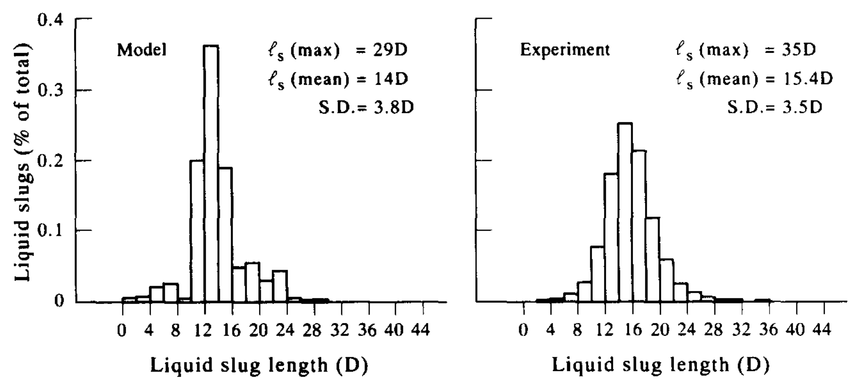

Figure 7. Slug length distribution at $x = 6$ m—comparison with experimental results; air–water, 5.0 cm dia, $U_{LS} = 0.01$ m/s, $U_{GS} = 0.25$ m/s.

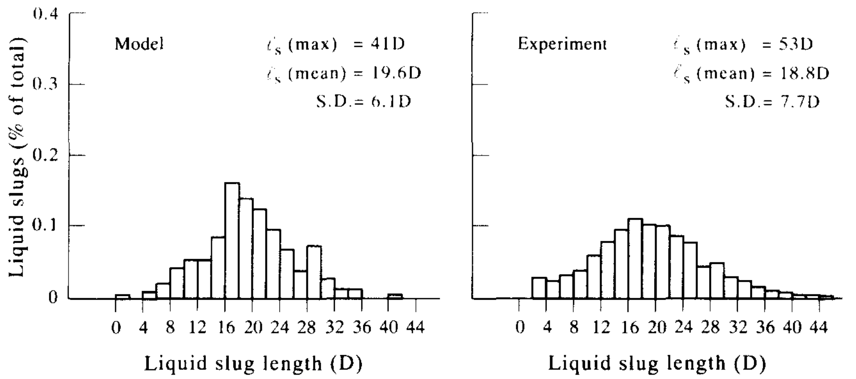

Figure 8. Slug length distribution at $x = 6$ m—comparison with experimental results; air–water, 5.0 cm dia, $U_{LS} = 0.75$ m/s, $U_{GS} = 0.75$ m/s.

RESULTS AND DISCUSSION

In figure 2 only 24 slugs are shown, out of which 17 slugs disappear within a short distance. To get meaningful statistics we introduced 10,000 slugs at the pipe entrance. At each predetermined position a count of the lengths of the slugs which pass that point is recorded. The calculated histograms of the slug length distribution at various positions along the pipe are shown in figures 3–6. All the results are for air–water flowing upward in a vertical pipe of diameter $D = 0.05$ m. Two sets of mixture velocities are illustrated. Figures 3 and 4 show typical results for a relatively low mixture velocity ($U_{LS} = 0.01$ m/s, $U_{GS} = 0.25$ m/s), while figure 5 and 6 are the results for high mixture velocity ($U_{LS} = 0.75$ m/s, $U_{GS} = 0.75$ m/s).

The instantaneous translational velocity was assumed to follow [3] with $B = 5.5$ and $\beta = 0.6$. These constants are our fit to Moissis & Griffith's (1962) results for a 5 cm pipe diameter. The terminal translational velocity U_{t_∞} was assumed to follow the conventional correlation [4] with the drift velocity $U_d = 0.35\sqrt{gD}$ (Dumitrescu 1943). As has been mentioned before, l_{stab} in [3] is the minimum length of a stable slug. The translational velocity of bubbles behind slugs longer than this minimum length is constant and equals U_{t_∞}. It has been shown (Shemer & Barnea 1987; Taitel & Barnea 1990) that the minimum stable slug length is related to the length of the liquid slug needed to reestablish the fully developed velocity profile. Based on Van Hout *et al.* (1992), the wake region of the liquid slugs is longer for higher mixture velocities: l_{stab} is about $10D$ for the case of low mixture velocity (figures 3 and 4) and $15D$ for the case shown in figures 5 and 6.

Although we do use [3] as a correlation for the instantaneous translational velocity, it should be mentioned that the exact functional dependence of the translational velocity on the operating conditions is still an open question. It is, however, outside the scope of this work.

In figures 3–6 the histograms of the slug length distribution are plotted at various locations, namely at $x = 0$ (the entrance), 6, 10, 15 and 20 m. The plots show the relative number of slugs within each interval of $1D$. The ordinate n_i/n was normalized with respect to the maximum value of n_i/n at $x = 0$, to ensure maximum visibility.

For the case in figure 3 the entrance distribution was introduced as a random uniform number between 2–$6D$. This is shown in figure 3 by the equal distribution of the number of slugs between 2–3, 3–4, 4–5 and 5–$6D$. At a distance of 2 m from the entrance, the number of slugs passing the station is reduced to 2176 out of the 10,000 slugs that entered the pipe, as a result of the merging process. At the location $x = 2$ m, the slug length distribution is already significantly different from the uniform distribution at the entrance. One can observe very short slugs (just before they disappear) as well as long slugs up to $22D$. At $x = 6$ m, this process is continued: the total number of slugs passing $x = 6$ m reduces to 1469 and the slug distribution is seen to contain fewer short slugs, some long slugs and a peak number of slugs between 12 to $13D$. At station $x = 10$ m, an almost fully developed situation is reached, while at $x = 15$ and 20 m the flow is way into the fully developed region. The number of slugs remains constant at 1377 (out of 10,000) and no slug is shorter than the minimum stable slug length $l_{stab} = 10D$.

In addition to the slug length distribution at each station, figure 3 also contains the values of the average slug length, the standard deviation (σ) and the maximum length of the liquid slug. For $x = 0$ the average length is $4D$ ($\sigma = 1.15D$) and the maximum slug length is $6D$. As one moves away from the entrance, the aforementioned variables increase and for the fully developed situation we obtain $l_s(\text{mean}) = 14.5D$ ($\sigma = 3.4D$) and $l_s(\text{max}) = 28.4D$.

In figure 4 we observe the results for the same conditions as in figure 3, the only difference being that the random entrance distribution follows a normal distribution with an average of $4D$ and standard deviation of $1D$. The results are very similar to those shown in figure 3, indicating a very small effect of the initial distribution.

Figures 5 and 6 are typical results for high mixture velocities, where l_{stab} was taken as $15D$. Here the fully developed distribution is obtained at distances longer than for the case shown in figures 3 and 4. For example, at station $x = 10D$ very short slugs are still seen. However, at these high mixture velocities the histograms are somewhat flatter with a larger standard deviation than in the case of low mixture velocities (figures 3 and 4).

The model results for slug length distribution were compared to the experimental results obtained by Van Hout *et al.* (1992). They sampled about 2500 slugs in vertical upward air–water flow for several flow conditions. Figure 7 shows the comparison for the case of low mixture velocity, while figure 8 compares the results for high mixture velocity. In both cases the model results compare fairly well with the experimental results, regarding the general shape of the distributions, the average slug length and the maximum slug length.

SUMMARY AND CONCLUSIONS

1. A model for the slug length distribution at various positions along the pipe has been developed. The model is based on the bubble overtaking mechanism that occurs due to the fact that the translational velocity of an elongated bubble behind short slugs is considerably higher than that behind long slugs.
2. Two kinds of slug length entrance distributions have been used, a random number set with a uniform distribution and a normal distribution. It has been shown that the evolution of the length distribution along the pipe, the fully developed distribution, the average slug length, the maximal slug length and the standard deviation are not sensitive to the slug length distribution at the pipe entrance.
3. The translational velocity of the elongated bubble, as a function of the liquid slug ahead of it, should be given as an input relation for this model. In the present work, the Moissis & Griffith (1962) relation for vertical flow has been used. Little work on the dependence of the bubble translational velocity on the slug length ahead of it has been published. We hope that this work will encourage future work on this particular subject.
4. The slug length distribution in the developed region seems to follow approximately the log normal shape. For fully developed flow all the slugs are longer than the minimum stable slug length, the mean slug length is about 1.5 times the minimum stable slug length and the maximum length seems to be about 3 times the minimum stable slug length.
5. The model also provides information on the length of the entry region needed to establish fully developed slug flow, and yields the distribution at any point in the developing zone.

REFERENCES

AKAGAWA, K. & SAKAGUCHI, T. 1966 Fluctuation of void ratio in two-phase flow (2nd report, Analysis of flow configuration considering the existence of small bubbles in liquid slugs). *Bull. JSME* **9**, 104–110.

BARNEA, D. & BRAUNER, N. 1985 Hold-up of the liquid slug in two phase intermittent flow. *Int. J. Multiphase Flow* **11**, 43–49.

BARNEA, D. & SHEMER, L. 1989 Void-fraction measurements in vertical slug flow: applications to slug characteristics and transition. *Int. J. Multiphase Flow* **15**, 495–504.

BERNICOT, M. & DROUFFE, J. M. 1989 Slug length distribution in two-phase transportation systems. In *Proc. 4th Int. Conf. Multi-Phase Flow*, Nice, France, pp. 485–493. BHRA, Cranfield, Beds.

BRILL, J. P., SCHMIDT, Z., COBERLY, W. A., HERRING, J. D. & MOORE, D. W. 1981 Analysis of two phase tests in large diameter flow lines in Prudhoe bay field. *Soc. Pet. Engng J.* **271,** 363–378.

DHULESIA, H., BERNICOT, M. & DEHEUVELS, P. 1991 Statistical analysis and modelling of slug lengths. In *Proc. 5th Int. Conf. On Multiphase Production*, Cannes, France, pp. 80–112. BHRA, Cranfield, Beds.

DUKLER, A. E. & HUBBARD, M. G. 1975 A model for gas–liquid slug flow in horizontal and near horizontal tubes. *Ind. Engng Chem. Fundam.* **14,** 337–347.

DUKLER, A. E., MOALEM-MARON, D. & BRAUNER, N. 1985 A physical model for predicting the minimum stable slug length. *Chem. Engng Sci.* **40,** 1379–1385.

DUMITRESCU, D. T. 1943 Stromung an einer luftblase im senkrechten rohr. *Z. Angew. Math. Mech.* **23,** 139–149.

FABRE, J. & LINÉ, A. 1992 Modeling of two-phase slug flow. *A. Rev. Fluid Mech.* **24,** 21–46.

FERNANDES, R. C. 1981 Experimental and theoretical studies of isothermal upward gas–liquid flows in vertical tubes. Ph.D. Thesis, Univ. of Houston, TX.

MOISSIS, R. 1963 The transition from slug to homogeneous two-phase flow. *J. Heat Transfer* **85,** 366–370.

MOISSIS, R. & GRIFFITH, P. 1962 Entrance effects in a two-phase slug flow. *J. Heat Transfer* **84,** 29–39.

NICHOLSON, M. K., AZIZ, K. & GREGORY, G. A. 1978 Intermittent two phase flow in horizontal pipes: predictive models. *Can. J. Chem. Engng* **56,** 653–663.

NYDAL, O. J., PINTUS, S. & ANDREUSSI, P. 1992 Statistical characterization of slug flow in horizontal pipes. *Int. J. Multiphase Flow* **18,** 439–453.

SAETHER, G., BENDIKSEN, K., MÜLLER, J. & FROLAND, E. 1990 The fractal statistics of liquid slug lengths. *Int. J. Multiphase Flow* **16,** 1117–1129.

SHEMER, L. & BARNEA, D. 1987 Visualization of the instantaneous velocity profiles in gas–liquid slug flow. *PhysicoChem. Hydrodynam.* **8,** 243–253.

TAITEL, Y. & DUKLER, A. E. 1977 A model for slug frequency during gas–liquid flow in horizontal and near horizontal pipes. *Int. J. Multiphase Flow* **3,** 585–596.

TAITEL, Y. & BARNEA, D. 1990 Two phase slug flow. In *Advances in Heat Transfer* (Edited by HARTNETT, J. P. & IRVINE, T. F. JR), Vol. 20, pp. 83–132. Academic Press, New York.

TAITEL, Y., BARNEA, D. & DUKLER, A. E. 1980 Modeling flow pattern transitions for steady upward gas–liquid flow in vertical tubes. *AIChE Jl* **26,** 345–354.

VAN HOUT, R., SHEMER, L. & BARNEA, D. 1992 Spatial distribution of void fraction within the liquid slug and some other related slug parameters. *Int. J. Multiphase Flow* **18,** 831–845.